DISPERSION RELATION OF THE RHO-MESON AT FINITE TEMPERATURE AND DENSITY

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Abstract

The ρ meson mass shift, width broadening, and spectral density at finite temperature and nucleon density are estimated using a general formula which relates the self-energy to the real and imaginary parts of the forward scattering amplitude on the constituents of the medium. We saturate the scattering amplitude at low energies with resonances, while at high energies a Regge approach is taken in combination with vector meson dominance; experimental data is used wherever possible. The main result is that the ρ meson becomes increasingly broad with increasing nucleon density. The spectral density is suppressed in the resonance region $600 < M < 900$ MeV and enhanced in the subresonance region $M < 600$ MeV.

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The problem of how the properties of hadrons change in hadronic or nuclear matter in comparison to their free space values has attracted a lot of attention. Among the properties of immediate interest are the in-medium particle's mass shift and width broadening. Different models, as well as model independent approaches, have been used to calculate these effects, both at finite temperature and finite density. It is clear on physical grounds that the in-medium mass shift and width broadening of a particle are only due to its interaction with the constituents of the medium, for not too dense media anyway. Thus one can use phenomenological information on this interaction to calculate the mass shift and width broadening: References [1, 2] give a few examples, reference [3] gives a relativistic field-theoretic derivation.

For meson a scattering from hadron b in the medium the contribution to the self-energy is:

$$
\begin{aligned}
\Pi_{ab}(E,p) &= -4\pi \int \frac{d^3k}{(2\pi)^3} n_b(\omega) \frac{\sqrt{s}}{\omega} f_{ab}^{(\mathrm{cm})}(s) \\
&= -\frac{1}{2\pi p} \int_{m_b}^{\infty} d\omega \, n_b(\omega) \int_{s_-}^{s_+} ds \sqrt{s} f_{ab}^{(\mathrm{cm})}(s) \,,
\end{aligned}
\tag{1}
$$

where E and p are the energy and momentum of the particle, $\omega^2 = m_b^2 + k^2$,

$$
s_{\pm} = E^2 - p^2 + m_b^2 + 2(E\omega \pm pk) \,,
\tag{2}
$$

n_b is either a Bose-Einstein or Fermi-Dirac occupation number, and f_{ab} is the forward scattering amplitude. The normalization of the amplitude corresponds to the standard form of the optical theorem

$$
\sigma = \frac{4\pi}{q_{\mathrm{cm}}} \mathrm{Im} f^{(\mathrm{cm})}(s) \,,
\tag{3}
$$

where q_{cm} is the momentum in the cm frame. The dispersion relation is determined by the poles of the propagator after summing over all target species and including the vacuum contribution to the self-energy:

$$
E^2 - m_a^2 - p^2 - \Pi_a^{\mathrm{vac}}(E,p) - \sum_b \Pi_{ab}(E,p) = 0 \,.
\tag{4}
$$

The applicability of eq. (1) is limited to those cases where interference between sequential scatterings is negligible.

Taking various limits of eq. (1) is instructive. First of all, we note that the cross section is invariant under longitudinal boosts. It is convenient to know how the scattering amplitude transforms. For the same relative velocity:

$$
m_a f_{ab}^{(a\,\mathrm{rest\,frame})} = m_b f_{ab}^{(b\,\mathrm{rest\,frame})} = \sqrt{s} f_{ab}^{(\mathrm{cm})} \,.
\tag{5}
$$

In the limit that the target particles b move nonrelativistically we can approximate ω in the first line of eq. (1) with m_b, in which case

$$
\Pi_{ab} = -4\pi f_{ab}^{(b\,\mathrm{rest\,frame})} \rho_b \,,
\tag{6}
$$

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where ρ_b is the spatial density. Next consider the chiral limit of pions serving as the target particles, relevant for low temperature baryon free matter. From eq. (5) $\sqrt{s}f_{a\pi}^{\rm (cm)} = m_a f_{a\pi}^{(a\,{\rm rest\,frame})}$. Since $f_{a\pi}^{(a\,{\rm rest\,frame})}$ involves two derivative couplings of the pion to the massive state a (Adler's theorem) one sees from eq. (1) that $\Pi_{a\pi} \sim T^4$. See also ref. [4]. (In contrast note that $\Pi_{\pi\pi} \sim T^2$.) Finally, if the self-energy is evaluated in the rest frame of a it is possible to do all the integrations but one.

$$\Pi_{ab}(E, p) = -\frac{m_a^2 T}{\pi p} \int_{m_b}^{\infty} d\omega \ln \left[\frac{1 - \exp(-\omega_+/T)}{1 - \exp(-\omega_-/T)} \right] f_{ab}^{(a\,{\rm rest\,frame})}(\omega) \qquad (7)$$

Here $\omega_\pm = (E\omega \pm pk)/m_a$. This assumes that b is a boson; a similar formula ensues if it is a fermion.

In this paper we will estimate the ρ meson dispersion relation for finite temperature and baryon density for momenta up to a GeV/c or so as this is very interesting for the production of dileptons in high energy heavy ion collisions [5]. Oftentimes such investigations use low energy effective Lagrangians which are matched to experimental data. Here we will saturate the low energy part of the scattering amplitude with resonances and use a combination of vector meson dominance (VMD) and Regge theory at high energy.

We will assume that ρ-mesons are formed during the last stage of the evolution of hadronic matter created in a heavy ion collision, when the matter can be considered as a weakly interacting gas of pions and nucleons. This stage is formed when the local temperature is on the order of 100 to 150 MeV and when the local baryon density is on the order of the normal nucleon density in a nucleus. The description of nuclear matter as a noninteracting gas of nucleons and pions, of course, cannot be considered as a very good one, so it is clear from the beginning that our results may be only semiquantitative. The main ingredients of our calculation are $\rho\pi$ and ρN forward scattering amplitudes and total cross sections.

As we mentioned already, the scattering amplitudes are saturated by resonances at low energies. At high energies we determine them with the aid of VMD: $\sigma_{\gamma N}$ is well known experimentally, $\sigma_{\gamma\pi}$ can be found by the Regge approach. Afterwards $Ref_{\rho N}$ and $Ref_{\rho\pi}$ are determined from dispersion relations. Since VMD allows one to find only the cross sections of transversally polarized ρ-mesons, we restrict ourselves to this case. As was shown in [2], for energies greater than 2 GeV the effects on longitudinal ρ-mesons in nuclear matter are much smaller than for transverse ones. At finite temperature they are comparable [6]. Therefore, our results should be multiplied by a factor ranging from 2/3 to 1 for unpolarized ρ-mesons. The actual construction of the cross sections and scattering amplitudes was reported in [7] to which we refer the interested reader.

We will consider the momentum p to be real and evaluate the scattering amplitudes on-shell, that is, evaluate the self-energy at $E = \sqrt{p^2 + m_\rho^2}$. In this case Eqs. (1) and (4) take the form

$$E^2 = m_\rho^2 + p^2 + \Pi_\rho^{\rm vac} + \Pi_{\rho\pi}(p) + \Pi_{\rho N}(p). \qquad (8)$$

3

Since the self-energy has real and imaginary parts so does $E(p) = E_R(p) - i\Gamma(p)/2$. In the narrow width approximation the dispersion relation is determined from

$$\begin{aligned}
E_R^2(p) &= p^2 + m_\rho^2 + \text{Re}\Pi_{\rho\pi}(p) + \text{Re}\Pi_{\rho N}(p) \,, \\
\Gamma(p) &= -\left[\text{Im}\Pi_\rho^{\text{vac}} + \text{Im}\Pi_{\rho\pi}(p) + \text{Im}\Pi_{\rho N}(p)\right]/E_R(p) \,.
\end{aligned} \tag{9}$$

The width of the ρ-meson in vacuum, $\Gamma_\rho^{\text{vac}} = -\text{Im}\Pi_\rho^{\text{vac}}/m_\rho$, is 150 MeV. We can also define a mass shift and optical potential in the usual way.

$$\begin{aligned}
\Delta m_\rho(p) &= \sqrt{m_\rho^2 + \text{Re}\Pi_{\rho\pi}(p) + \text{Re}\Pi_{\rho N}(p)} - m_\rho \,, \\
U(p) &= E_R(p) - \sqrt{m_\rho^2 + p^2} \,.
\end{aligned} \tag{10}$$

We shall evaluate these for temperatures of 100 and 150 MeV and nucleon densities of 0, 1 and 2 times normal nuclear matter density (0.155 nucleons/fm^3). This is accomplished by utilizing a Fermi-Dirac distribution for nucleons. The nucleon chemical potentials are 745 and 820 MeV for densities of 1 and 2 times normal at $T = 100$ MeV, and 540 and 645 MeV for densities of 1 and 2 times normal at $T = 150$ MeV. Anti-nucleons are not included.

Here we would like to make a trivial point that is nevertheless not discussed much in the literature. In our first paper [7] we defined the width in the rest frame of the ρ-meson. In this paper we define the width in the rest frame of the thermal system. The former definition is conventional and most useful in particle physics; the latter definition is the usual one in statistical and many-body physics, whether the system be nonrelativistic or relativistic. Either definition is equally valid. For example, consider the latter definition in the limit of a vanishingly small density. The width becomes $\Gamma(p) = (m_\rho/E)\Gamma_\rho^{\text{vac}}$ which decreases with increasing momentum. This is just the time dilation effect and has nothing to do with the ρ-meson moving through a many-particle system.

Figure 1 shows the mass shift at temperatures of 100 and 150 MeV and for nucleon densities of 0, 1 and 2 times normal density. The effect with pions alone is negligible (on the order of 1 MeV). The main effect comes from nucleons. The effective mass increases with nucleon density and with momentum, but is almost independent of temperature. At zero momentum the mass shift is about 15 MeV, reaching about 55 MeV at a momentum of 1 GeV/c for 1 times nucleon density. For 2 times nuclear density these mass shifts are about 30 and 110 MeV, respectively. These trends and numbers are roughly consistent with other analyses [9, 10, 11].

Similar trends occur in the ρ meson potential as may be seen in figure 2. The potential is positive, is on the order of tens of MeV, and increases with density.

Figure 3 shows the behavior of the ρ meson width with temperature, density, and momentum. Once again pions have very little effect. The main effect comes from nucleons. Contrary to ρ mesons moving in vacuum or through a pure pion gas the width remains roughly constant with momentum when nucleons are present. The width is about 240 MeV

at 1 times nuclear density and about 370 MeV at 2 times nuclear density. This means that the ρ meson becomes a rather poorly defined excitation with increasing nucleon density.

The rate of dilepton production is directly proportional to the imaginary part of the photon self-energy [12, 13] which is itself proportional to the imaginary part of the ρ meson propagator because of vector meson dominance [8, 6].

$$E_+ E_- \frac{dR}{d^3 p_+ d^3 p_-} \propto \frac{-\text{Im}\Pi_\rho}{[M^2 - m_\rho^2 - \text{Re}\Pi_\rho]^2 + [\text{Im}\Pi_\rho]^2} \tag{11}$$

The vacuum part of Π can only depend on the invariant mass, $M^2 = E^2 - p^2$, whereas the matter parts can depend on E and p separately. However, in the approximation we are using the scattering amplitudes are of necessity evaluated on the ρ meson mass shell. This means that the matter parts only depend on p because M is fixed at m_ρ. In particular, the imaginary part of the matter contribution does not vary with M. Note that, in general, it need not vanish until the one pion threshold is reached because it is a scattering process, not a decay. The vacuum parts can be obtained from the Gounaris-Sakurai formula [8, 6]. This formula gives a very good description of the pion electromagnetic form factor, as measured in $e^+ e^-$ annihilation [14], up to 1 GeV apart from a small mixing with the ω meson which we are ignoring in this paper.

$$\begin{aligned}
\text{Re}\Pi_\rho^{\text{vac}} &= \frac{g_\rho^2 M^2}{48\pi^2} \left[\left(1 - 4m_\pi^2/M^2\right)^{3/2} \ln \left| \frac{1 + \sqrt{1 - 4m_\pi^2/M^2}}{1 - \sqrt{1 - 4m_\pi^2/M^2}} \right| + 8m_\pi^2 \left(\frac{1}{M^2} - \frac{1}{m_\rho^2} \right) \right. \\
&\quad \left. - 2 \left(\frac{p_0}{\omega_0} \right)^3 \ln \left(\frac{\omega_0 + p_0}{m_\pi} \right) \right] \tag{12} \\
\text{Im}\Pi_\rho^{\text{vac}} &= -\frac{g_\rho^2 M^2}{48\pi} \left(1 - 4m_\pi^2/M^2\right)^{3/2} \tag{13}
\end{aligned}$$

Here $2\omega_0 = m_\rho = 2\sqrt{m_\pi^2 + p_0^2}$. The vacuum width is $\Gamma_\rho^{\text{vac}} = (g_\rho^2/48\pi)m_\rho(p_0/\omega_0)^3$.

The imaginary part of the propagator is directly proportional to the spectral density. The former is plotted in figure 4 for a pure pion gas at a temperature of 150 MeV and in figure 5 for pions and nucleons at a density of 1 times nuclear and a temperature of 100 MeV. These parameters are characteristic of the final stages of a high energy heavy ion collision. Pions alone have very little effect on the spectral density even at such a high temperature. The effect of nucleons, however, is dramatic. The spectral density is greatly broadened, so much so that the very idea of a ρ meson may lose its meaning.

The above observations and remarks on the relative importance of pions and nucleons may need to be re-examined when really applying these calculations to heavy ion collisions. The pions may be overpopulated in phase space, compared to a thermal Bose-Einstein distribution, and this could be modeled either by introducing a chemical potential for pions or simply by multiplication by an overall normalization factor. Pions would need

to be enhanced by a substantial factor (5 or more) to make a noticeable contribution at a density of 0.155 nucleons per fm^3.

Recently preliminary data in Pb-Au collisions at 160 GeV·A have been presented [15] where, in studying the e^+e^- mass spectrum, it was found that the ρ-peak is absent at $k_T(e^+e^-) < 400$ MeV, but reappears as a broad enhancement at $k_T(e^+e^-) > 400$ MeV. This appears to be just the opposite of our findings. However, our calculations refer to the ρ momentum relative to the local rest frame of the matter. In a heavy ion collision, the matter generally flows outward from the central collision zone. Therefore a low momentum ρ meson may actually be moving faster relative to the outflowing matter than a higher momentum one. No conclusion can really be drawn without putting our results into a space-time model of the evolution of matter in a heavy ion collision.

In summary, we have studied the properties of the neutral ρ meson in a finite temperature pion gas with and without nucleons present. Nucleons play the dominant role. They provide a generally positive potential for the ρ mesons and greatly increase their width. The ρ meson spectral density is so broadened that the ρ may lose its identity as a well defined particle or resonance. Our results are based on experimental information on the scattering amplitudes and as such provide a direct extrapolation from zero temperature and density to nonzero values of both. At sufficiently high energy density the matter can no longer be described very well as a gas of noninteracting pions and nucleons. Nevertheless the trends must be obeyed by any realistic calculations of the ρ meson in-medium. Applications to thermal and hydrodynamic models of heavy ion collisions are under investigation.

Acknowledgments

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Figure Captions

Figure 1: The ρ meson mass shift as a function of momentum. The dashed lines represent $T = 100$ MeV, the solid lines represent 150 MeV. The density of nucleons is indicated.

Figure 2: The ρ meson potential as a function of momentum. The dashed lines represent $T = 100$ MeV, the solid lines represent 150 MeV. The density of nucleons is indicated.

Figure 3: The ρ meson width as a function of momentum. The dashed lines represent T = 100 MeV, the solid lines represent 150 MeV. The density of nucleons is indicated.

Figure 4: The imaginary part of the ρ meson propagator as a function of invariant mass for fixed values of momentum as indicated. The temperature is 150 MeV and the matter is free of nucleons.

Figure 5: The imaginary part of the ρ meson propagator as a function of invariant mass for fixed values of momentum as indicated. The temperature is 100 MeV and the nucleon density is the same as in ordinary nuclei: 0.155 nucleons/fm^3.

Im D (MeV^{-2})

vacuum
T=150 MeV, p=0
T=150 MeV, p=1 GeV

n=0

M (MeV)